Grade 7/8 Math Circles
Week of 24th October
BCC Prep

BCC

The BCC is a problem-solving contest with a focus on computational and logical thinking. Questions are inspired by topics in computer science but only require comfort with concepts found in mathematics curriculum common to all provinces. Connections to Computer Science are described in the solutions to all past contests.

Students in grade 8 or below can write the Grade 7/8 BCC. The Grade 7/8 BCC consists of 15 multiple choice questions divided into 3 parts with 90 marks total: 5 questions in Part A worth 8 marks each, 5 questions in Part B worth 6 marks each, and 5 questions in Part C worth 4 marks each. For a maximum of 6 questions, it is possible to receive 2 marks for every unanswered question. Students are given exactly 45 minutes to answer the questions. Some calculators are permitted. Each question on the BCC is given by a story and a question. The story provides the background information required to solve the question

Some more info!
You can find more info about the contest here: https://cemc.uwaterloo.ca/contests/bcc
You can find previous competitions and solutions here: https://www.cemc.uwaterloo.ca

Strategies

There are a few strategies that you should implement when approaching these types of logical and computing questions.

- First read the story, then the question, then reread the story. This will help with finding the details needed to solve the question as well as understanding what the question is asking.
- Underline or write down the important information in the story and question. Split the problem up into multiple parts or steps if it is long. Connect them at the end of your process.
- Rule out answers that are impossible or that you can show aren't the solution. The problems are all multiple choice and will have 4 options, so ruling out a couple incorrect answers can

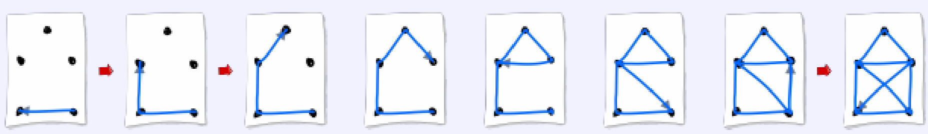
help with deciding on the correct answer. When in doubt, make a logical guess.

- Have fun writing the contest! This contest is meant to be an enjoyable experience that will motivate your interest in math and computer science. The BCC emphasizes participation rather than competition, so be proud of trying.

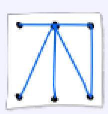
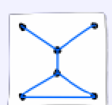
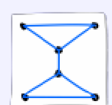
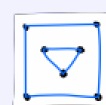
BCC Prep Questions

Problem 1: *Connect the Dots* (2020)

Zhi likes to draw. He creates his pictures by drawing dots and then connecting them with line segments in one motion, never picking up his pencil and never drawing the same line segment twice. For example, this is how Zhi draws a picture of a house:

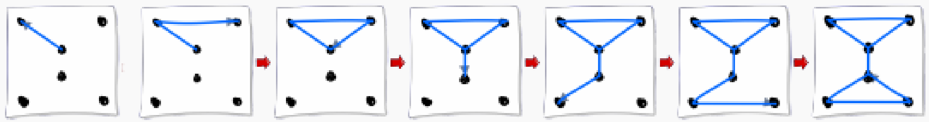


Which of the following can Zhi draw

(A)  (B)  (C)  (D) 

Answer 1

Zhi can only draw option C). This is how he would do it



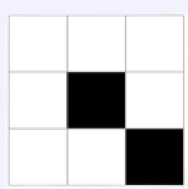
It is important to understand why options A), B), and D) are impossible to draw. Since a line segment cannot be drawn twice, any dot that joins two line segements must be where Zhi ends his drawings. Since in option A), there are more than two dots joined to exactly one line segment, Zhi cannot draw this diagram. Likewise, to draw Option B), Zhi must begin drawing at one of the top two dots and finish drawing at the other of the two top dots. This forces him to draw the line segment in the middle twice, therefore Zhi cannot draw this diagram. Finally, Option D consists of two disconnected pieces so there is no way for Zhi to draw this picture without picking up his pencil.

Problem 2: ***QB-Code*** **(2015)**

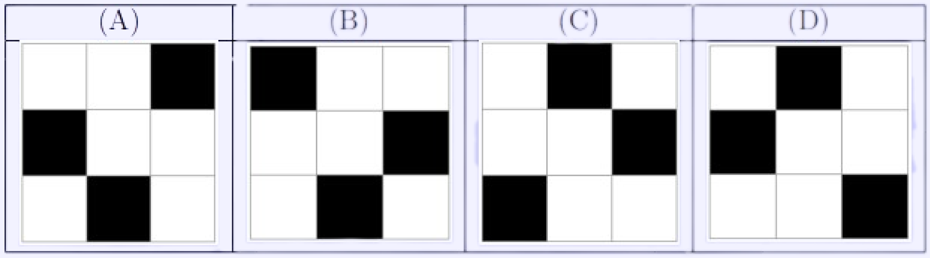
Beavers want to encode numbers for keeping track of how many trees they have chewed down. Therefore they developed the Quick-Beaver-Code (QB-Code). This is a graphical code consisting of nine 1×1 squares arranged into a 3×3 square. Every square has a certain value. The squares are filled line by line from the bottom to the top, from right to left. The next square has double the value of the square before. In the example, you see the values of the first five squares

...	...	...
...	16	8
4	2	1

To encode a number, the beavers darken some squares. The number encoded is the sum of the values of the dark squares. For example, the number encoded in this QB-Code is 17:



Which of the following encodes the largest number?



Answer 2

Option B) encodes the largest number. We know this without any complicated computations. The top left square, 256 (can be found by calculating 2^8) is greater than the sum of ALL the other squares ($1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255 < 256$). So, any option that has the top left square coloured will automatically be the largest value encoded.

Problem 3: *Beavers vs. Dogs* (2015)

Beavers and dogs compete. The nine participants scored the following points: $1, 2, 2, 3, 4, 5, 5, 6, 7$

We know that no dog scored more than any beaver, but one dog had the same score as a beaver and two dogs also had the same score. How many dogs took part in the competition

(A) 2 (B) 3 (C) 6 (D) 7

Answer 3

There are (C) 6 dogs that took part into the competition. If no dog scored more than any beaver, we can order the animals in a row so that a separator can be used to separate the dogs and beavers.

$$\text{(Dogs) } 1, 2, 2, 3 \mid 4, 5, 5, 6, 7 \text{ (Beavers)}$$

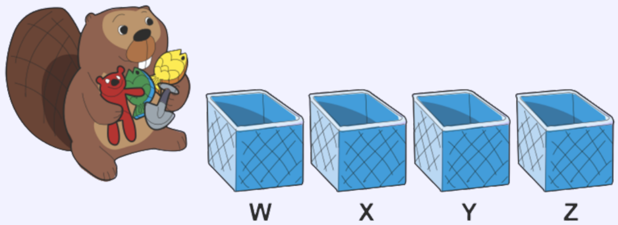
We denote $\mid$ to be the separator between dogs and beavers. If two dogs scored 5, then since all dogs scored less than beavers, then two dogs must have also scored 2. This, however, does not allow for the fact that a dog and a beaver tied. Hence, the two 5's must be a dog and a beaver, which means that the separator between the dogs and the beavers must be between the two 5's:

$$\text{(Dogs) } 1, 2, 2, 3, 4, 5 \mid 5, 6, 7 \text{ (Beavers)}$$

Notice that if the separator between dogs and beavers was between the 2's, two dogs must have scored 5 and 5 and then they will be better than the beaver that scored 2. This is in contrary to the task statement, so it cannot be possible. Therefore, 6 dogs participated in the competition.

Problem 4: *Triple Trouble* (2019)

A beaver puts each of four toys into boxes labeled W, X, Y, and Z. Each box can hold any number of toys.



At least one of the three conditions in each row of the table shown is satisfied.

a toy is in X	no toy is in Y	no toy is in Z
a toy is in W	a toy is in X	no toy is in Z
no toy is in X	no toy is in Y	a toy is in Z
no toy is in W	no toy is in X	no toy is in Y
no toy is in X	a toy is in Y	no toy is in Z

What is the minimum possible number of empty boxes?

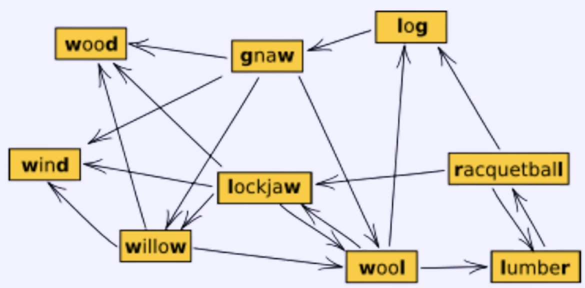
(A) 3 (B) 2 (C) 1 (D) 0

Answer 4

The minimum possible number of empty boxes is (C) 1. Notice that if there is a toy in each box, then no condition in the fourth row is satisfied. This means there must be at least one empty box. To show that it is possible to have exactly one empty box, consider placing a toy in boxes X, Y, and Z. (We could find this combination by trying the four possible combinations of three boxes.) Since a toy is in box X, the first two rows have a condition satisfied. Since a toy is in box Z, the third row has a condition satisfied. Since there is not a toy in box W, the fourth row has a condition satisfied. Finally, since there is a toy in box Y, the fifth row has a condition satisfied. We have shown that the minimum possible number of empty boxes is one.

Problem 5: *Longest Word Chain* (2018)

Beavers play a word chain game. One beaver starts by saying a word. The other beaver must say a different word which begins with the last letter of the previous word. Then the first beaver says another word (which was not said yet) using this same rule, and so on. If a beaver is unable to say a new word, that beaver loses the game. These beavers do not know many words. In fact, they can draw their entire vocabulary like this:



Notice that an arrow out of a word points at the next possible word(s) that can be said.

(A) 6 (B) 7 (C) 8 (D) 9

Answer 5

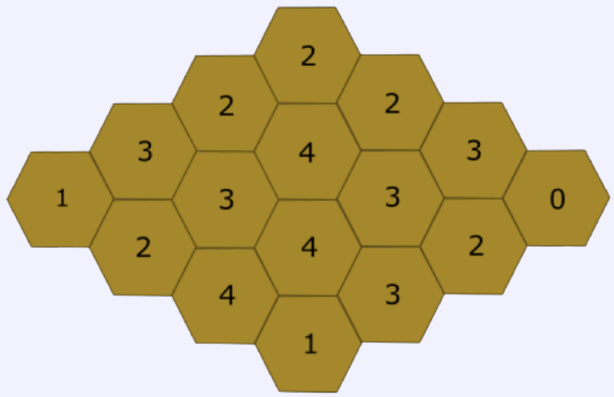
The beavers can use at most (C) 8 words in one game. One example is:

lockjaw-wool-lumber-racquetball-log-gnaw-willow-wood

(Can you find another game of the same length?) To be sure that 8 is the largest possible number of words, we have to convince ourselves that it is not possible to use all 9 words. Consider the words wood and wind. There is no word beginning with d, so if either of these words is said, it must be the last word of the game. Since there cannot be two words that are said last, it is not possible to use the entire vocabulary of 9 words.

Problem 6: *Beehive* (2017)

A bear studies how many hexagons in a honeycomb contain honey. For each hexagon, the bear records how many other hexagons touching this hexagon contain honey. So this number could be $0, 1, 2, 3, 4, 5$ or 6. The results of the bear's study are below.

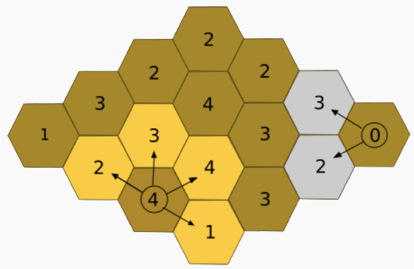


How many hexagons contain honey?

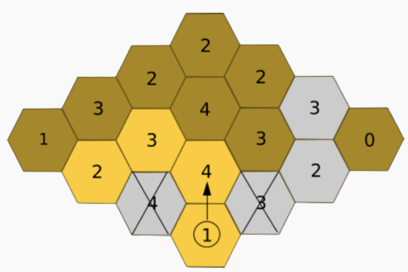
(A) 7 (B) 8 (C) 9 (D) 10

Answer 6

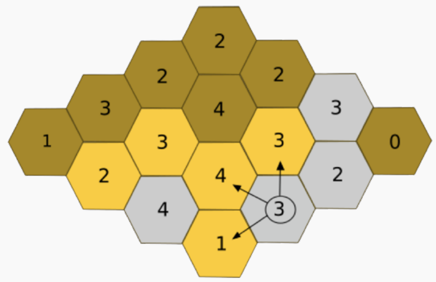
There are (C) 9 hexagons that contain honey. One way to solve this task is to start from the cells for which the content of their adjacent cells is known. It is the case for two cells, that have a circle in the picture below: All the adjacent cells of the cells with a 0 are empty All the adjacent cells of the cells with a 4 are full We will use yellow to represent honey in a hex, grey to represent no honey, and brown for undetermined.



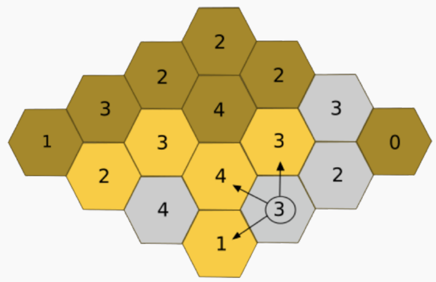
After that, we can easily find some others cells, near the zone that we have changed, for which the content of their adjacent cells is now known. For instance the cell with a circle in the picture below, has only 1 full adjacent cell. It is the one already colored in yellow, just above it. So its other adjacent cell is empty.



Then, the cell with a 3 on it, which we just coloured grey, must have exactly 3 full adjacent cells. We have already two of them on its left, the one on the right is empty, so the one above it must be full.

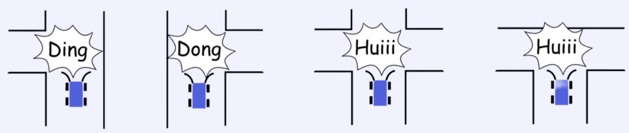


We can go on from one adjacent cell to the others, in order to cover all the beehive.

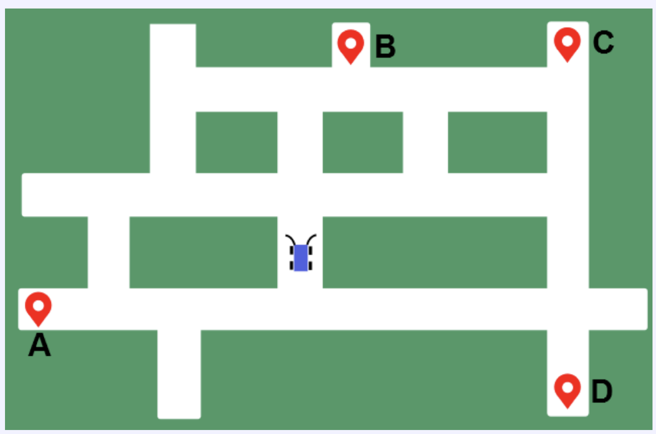


Problem 7: *Robotic Car* (2015)

Beavers have developed a robotic car. It has sensors that detect intersections. It produces the sounds shown below, when it is possible to turn left, right or both directions. The robotic car can go straight through an intersection (when possible), turn right (when possible) or turn left (when possible). The robotic car cannot make U-turns and cannot reverse.



It automatically stops when it senses an obstacle in front of it. The car drives around the map shown below, starting at the indicated position. As it drives around the map, it produces the sounds **Huiii Ding Huiii Dong**, in that order.

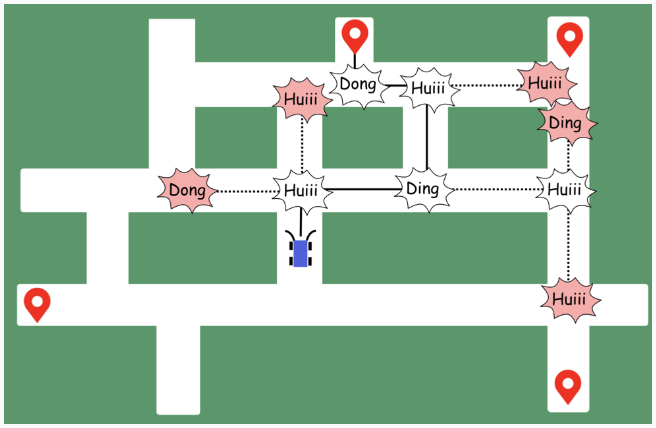


At which location does the car stop?

(*A*) Location A (*B*) Location B (*C*) Location C (*D*) Location D

Answer 7

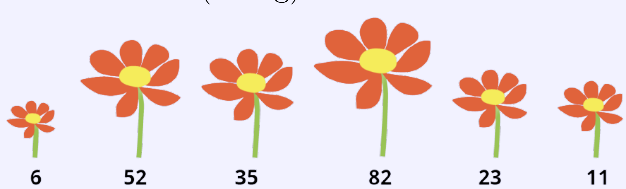
The image below shows the one and only route that the robotic car must take while producing the given sequence of sounds:



Notice that we try each possible direction at each intersection, and if the sound produced is not the correct/expected one, we backtrack to the previous intersection and try a different path.

Problem 8: ***Collecting Pollen*** **(2015)**

Beever the bee flies to a field of flowers to collect pollen. On each flight, he visits only one flower and can collect up to 10 mg of pollen. He may return to the same flower more than once. The initial amount of pollen in each flower (in mg) is shown below.



What is the maximum total amount of pollen that Beever can collect in 20 flights?

(A) 179 mg (B) 195 mg (C) 196 mg (D) 200 mg

One approach Beever could take is to collect as much pollen per flight as possible. This begins with Beever collecting 10 mg of pollen per flight while he can. We use division to calculate how many times he can do this:

$$\begin{aligned}
6\ mg &= 0 \cdot 10\ mg + 6\ mg \\
52\ mg &= 5 \cdot 10\ mg + 2\ mg \\
35\ mg &= 3 \cdot 10\ mg + 5\ mg \\
82\ mg &= 8 \cdot 10\ mg + 2\ mg \\
23\ mg &= 2 \cdot 10\ mg + 3\ mg \\
11\ mg &= 1 \cdot 10\ mg + 1\ mg
\end{aligned}$$

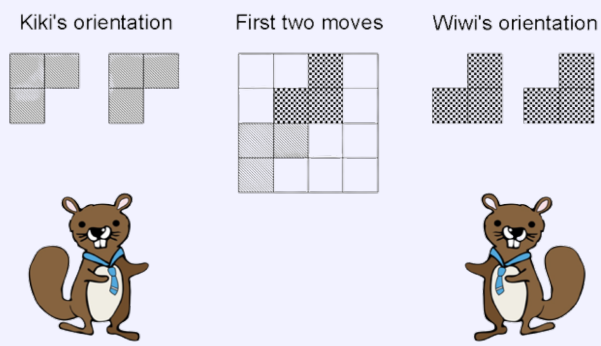
After $(0 + 5 + 3 + 8 + 2 + 1) = 19$ flights, he collects $19 \cdot 10\ mg = 190\ mg$ of pollen. In his 20th and final flight, Beever collects the largest amount left over, which is $6\ mg$. In total, Beever collects $19 \cdot 10\ mg + 6\ mg = 196\ mg$ of pollen. Notice that making any trip without taking the maximum will yield a total of less than $196\ mg$. Notice that once Beever decides how much pollen to collect on each flight, the order in which the flights happen does not matter. That is, we may take $6\ mg$ from the flower with $6\ mg$ of pollen on any trip, so long as we take $10\ mg$ from each of the other flights.

Problem 9: *L-Game* (2016)

Kiki and Wiwi are playing L-Game on a 4x4 board. The player who can no longer play a piece loses. They take turns placing L-shaped pieces one at a time with Kiki playing first so that

- every piece placed by Kiki is oriented as shown below,
- every piece placed by Wiwi is oriented as shown below,
- every piece is placed entirely on the board, and
- no two pieces overlap.

The diagram below illustrates a possible board after each player has placed a piece once.

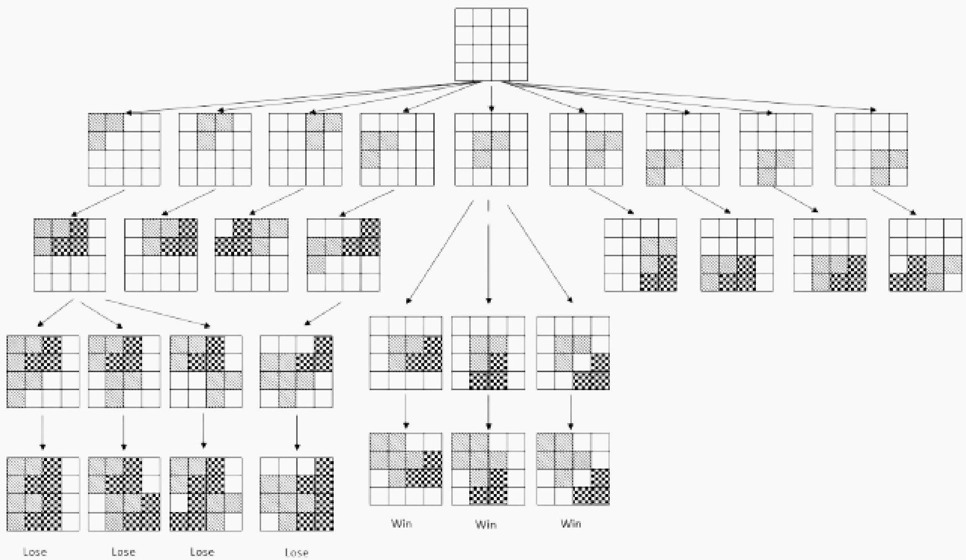


Starting from an entirely empty board, how many of Kiki's nine possible first moves guarantee that Kiki will win no matter what?

(A) 0 $\quad$ (B) 1 $\quad$ (C) 2 $\quad$ (D) 3

Answer 9:

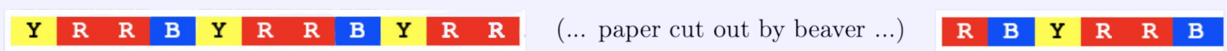
The correct answer is B. By placing a piece in the middle position, Kiki is guaranteed to win the game. No matter how Wiwi places a piece on his or her first turn, Kiki can only place a piece in the top left corner on her second turn. Then Wiwi cannot place a piece according to the rules. If Kiki places a piece in any other position on his/her first turn, then there is always at least one way that he/she can lose the game. The following diagram details some of the possibilities and symmetry can be used to rule out many of the positions.



Problem 10: *Party Banner* (2016)

You have a long roll of coloured paper for a party you are hosting. The paper consists of the following pattern repeated more than once:

A beaver cut out a section of the paper between pieces of length 11 and 6 as shown below.



Which of the following can be the length of the paper cut out by the beaver?

(A) 31 (B) 32 (C) 33 (D) 34

Answer 10:

The length is (A) 31. The first piece of the paper ends with YRR, meaning that the beaver has cut out at least one B. After that, it may have cut out any number of sequences of YRRB. The right side of the cut out paper must end with YR, since the second piece begins with RB. So, the length of her piece of paper is 1 (for the first B) plus $4 \cdot X$ (where X is the number of repeated YRRB patterns) plus 2 (for the YR) giving $4X + 3$ as the total length of her paper. Looking at the possible answers, we wish to determine which answer when subtracted by 3 gives a multiple of 4. A quick check shows that $31 - 3 = 28$ and $28 = 4 \cdot 7$. None of the other answers give a multiple of 4 after subtracting 3. Alternatively, we can notice that the length must be a multiple of 4. We know the lengths of the remaining pieces are 11 and 6, which adds to 17. We can try all four possible answers and determine that only $17 + 31 = 48$ gives a multiple of 4.